# THE GEOMETRY OF SCRAP PAPER

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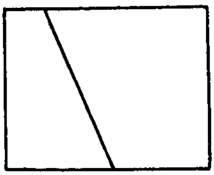
There is a great deal of geometry which students of all ages can experience and enjoy. Active geometry which draws student involvement in the process is something they not only enjoy, but will retain and use throughout their lives.

Since this is an activity-based geometry article, the reader must begin only after securing multiple sheets of $8^1/_2$ x 11 scrap paper.

## I A. Fold to form a line segment.

Take a sheet of paper and crease it.

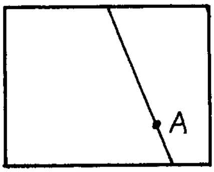


Is this: (a) a line?
(b) a line segment?

Indeed, the crease is a segment of a unique line (assuming the paper is not moved).

## I B. Fold to form a perpendicular.

With a pencil, draw a point on the crease (line segment) and label it point A.



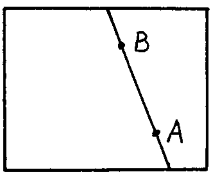

A new crease can be made which is perpendicular to the original crease at point A by ______________________________.

Unfortunately, this is not a trivial statement to complete for many people. Asking this question of middle school students requires a long wait time before anyone will respond. Then that response is most apt to be, "Fold the paper at point A?"

Folding the paper at point A so that the crease folds over itself creates the perpendicular line. That it is perpendicular is easy to "prove" by folding along the original crease to overlay each of the four angles which were formed. This activity provides an excellent opportunity to relate perpendicular, right angles, 90 degree angles and equal angles.

**I C. Fold to form a perpendicular bisector.**

Crease a piece of paper and place two points A and B on the crease.




Another crease perpendicular to the original crease and equidistant from points A and B can be made by ________________________________.

"By placing point A on point B and folding the line over itself." The new line is perpendicular because the line is folded over itself and goes through the midpoint of line segment AB (bisects AB). This line is the <u>perpendicular bisector</u>.

**II A. Compare angles directly.**

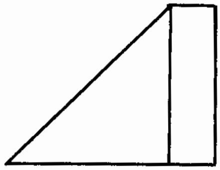
Take another sheet of paper and, without creasing, check to be sure that the angles at each corner are equal. Place them one over another. Are they equal?

**II B. Compare lengths directly.**

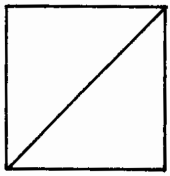
Are the opposite sides equal? Check without creasing.

## II C. Fold to form a square.

Are the adjacent sides equal? Obviously not. Place one side on the other beginning at a corner (vertex). Crease. Trim the excess to get a square.



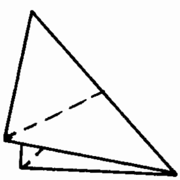
Opening the square, notice that the crease is a line segment joining opposite corners (vertices).



Third grade students know the name of this line, just not in this situation. Moving across the room, stopping and pointing at 45 degrees to your path while saying, "If I were walking down the sidewalk and did not go straight across at the corner but went that way, how would I be going?" The term diagonally will be elicited, and diagonal is the word you want.

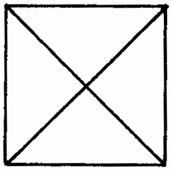
## II D. Fold to form the perpendicular bisector of a diagonal.

Place the vertices at the ends of the diagonal together and crease to obtain the perpendicular bisector of the diagonal.



Open. The crease is the other ______________________________.

A diagonal of a square is the perpendicular bisector of the other diagonal.



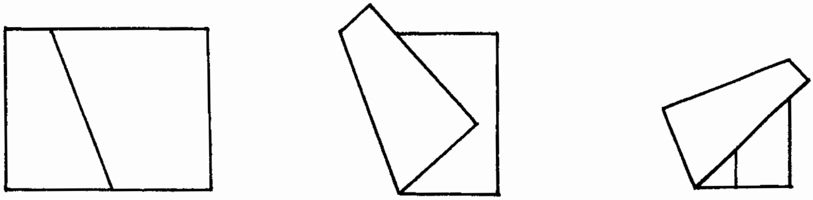
**II E. Compare angles bisected by diagonals.**

Fold the square along the diagonal. Notice that the diagonal has divided the angles into two equal angles (bisected the angle).

If the diagonals of a square are perpendicular bisectors of one another, is the figure a square?

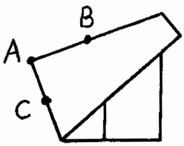
**III A. Form perpendicular folds.**

Take a new sheet of paper. Fold it. Leaving it folded, fold the crease back over itself to form a perpendicular crease.



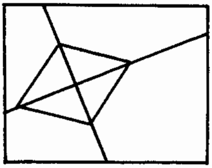
**III B. Choose vertices of a rhombus.**

Two of the edges are creased. Put marks B and C on each top crease at different distances from A.




Make a crease from B to C. When the sheet is opened, the diagonals are perpendicular bisectors of one another but the figure is not a square.

**III C. Fold to form a rhombus.**



It (the new figure) is a rhombus.

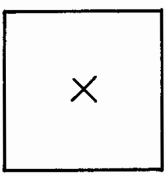
**III D. Compare sides of a rhombus.**

Is a rhombus equilateral?
Do the diagonals bisect the angles?
Fold the rhombus along each diagonal to see if the sides are equal.

The final sequence of activities leads to an illustration of the Pythagorean Theorem, $c^2 = a^2 + b^2$. Teachers speak of it mystically and kids know that it means lots of multiplication and large numbers. It has little or no other meaning.

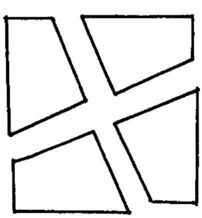
**IV A. Find the center of a square.**

Make an $8^1/_2$ x $8^1/_2$ square by using a second sheet to measure the length. Locate the center, being careful to crease the paper only at the center.



**IV B. Fold perpendicular lines through center.**

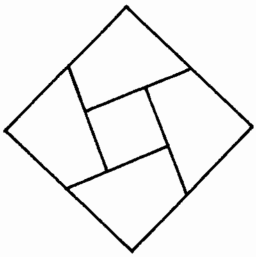
Fold a line through the center, which is not a diagonal, and crease the paper. Fold the paper back and forth several times so it will tear easily (later). Open the square and fold the perpendicular bisector. Fold the new crease back and forth and tear along both creases.



## IV C. **Rearrange pieces to form larger square.**

Rotate the pieces to note that the four pieces are the same size and shape (congruent).

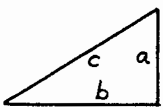
Remember that each piece has two right angles. If they are again rotated so that the right angles from the center of the original square become the corners, another larger square with a square hole at its center will result (provided the creases did not go through the vertices of the original square).



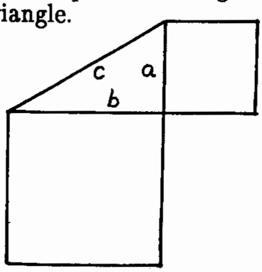
This procedure allows a method of making one large square from two smaller squares. The larger of the two squares is folded and torn or cut to fit around the smaller square.

## V A. **Construct squares to fit a right triangle.**

On a clean sheet of paper make marks on two adjacent sides which are closer to the vertex than $8^1/_2$ ". Crease from one point to the other and cut out the right triangle. Label the longest side (hypotenuse) c. Label the other sides a and b.



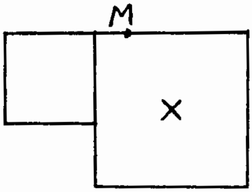

Make squares of length a and b. Place them appropriately next to the triangle.



The areas of the squares are $a^2$ and $b^2$. In the next activity you will combine the "a" square and the "b" square to make a "c" square which exactly fits along side c.

**V B. Construct two more matching squares.**

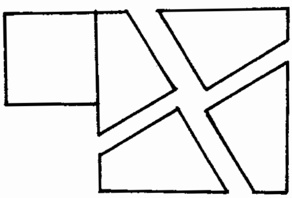
Make an additional "a" square and an additional "b" square. Find the center of the larger square ("b" square).




Place the two squares next to each other and locate the midpoint, M, of the line segment along their combined length.

Fold the "b" square through this point and its center. Open it and fold the perpendicular bisector.

**V C. Divide larger square as in IV B.**



**V D.** Rearrange pieces to show Pythagorean relation.

Cut square "b" and rotate the pieces to fit around "a" square making a new square, "a" square + "b" square. Now move it to side "c" to verify that it is "c" square.